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**STABILITY ANALYSIS OF DISTRIBUTED ENGINE  
CONTROL SYSTEMS UNDER COMMUNICATION  
PACKET DROP (POSTPRINT)**

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**Structures and Controls Branch  
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<b>14. ABSTRACT</b> Currently, Full Authority Digital Engine Control (FADEC), based on a centralized architecture framework is being widely used for gas turbine engine control. However, current FADEC is not able to meet the increased burden imposed by the advanced intelligent propulsion system concepts. This has necessitated development of the Distributed Engine Control system (DEC). FADEC based on Distributed Control Systems (DCS) offers modularity, improved control systems prognostics and fault tolerance along with reducing the impact of hardware obsolescence. Some of the challenges to be dealt with are selection of communication architecture, high temperature electronics and logical functional partitioning of centralized controller.					
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# Stability Analysis of Distributed Engine Control Systems Under Communication Packet Drop

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Currently, Full Authority Digital Engine Control (FADEC), based on a centralized architecture framework is being widely used for gas turbine engine control. However, current FADEC is not able to meet the increased burden imposed by the advanced intelligent propulsion system concepts. This has necessitated development of the Distributed Engine Control system (DEC). FADEC based on Distributed Control Systems (DCS) offers modularity, improved control systems prognostics and fault tolerance along with reducing the impact of hardware obsolescence. Some of the challenges to be dealt with are selection of communication architecture, high temperature electronics and logical functional partitioning of centralized controller. In this paper, we propose Decentralized Distributed Full Authority Digital Engine Control (D<sup>2</sup>FADEC) based on a two level decentralized control framework. The effect of decentralized controller on system stability and robustness through the Packet Dropping Margin (PDM) has been studied. A design method is proposed to decouple the centralized controller into different subsystems, thus reducing the effect of interactions between the subsystems. It is shown that PDM is largely dependent on the closed loop controller structure; hence use of a controller in a decentralized framework improves the PDM. Also, a F100 gas turbine engine example is used to show that PDM can be improved by partitioning the centralized system.

## Nomenclature

FADEC	= Full Authority Digital Engine Control
D <sup>2</sup> FADEC	= Decentralized Distributed Full Authority Digital Engine Control
DEC	= Distributed Engine Control
DCS	= Distributed Control Systems
NCS	= Networked Control Systems
DECWG	= Distributed Engine Control Working Group
PDP	= Packet Dropping Probability
PDM	= Packet Dropping Margin
$\xi$	= Independent and Identically Distributed (i.i.d.) Bernoulli random process
$\rho(\cdot)$	= Spectral radius of a matrix
$\otimes$	= Kronecker product
$\mu$	= $\lambda_{max}^+$ , where $\lambda^+$ are eigenvalues having positive real part
$\ \cdot\ _2$	= Spectral norm of a matrix
$K_2(\cdot)$	= Spectral condition number of a matrix
$(\cdot)^\#$	= Moore-Penrose Inverse of a matrix

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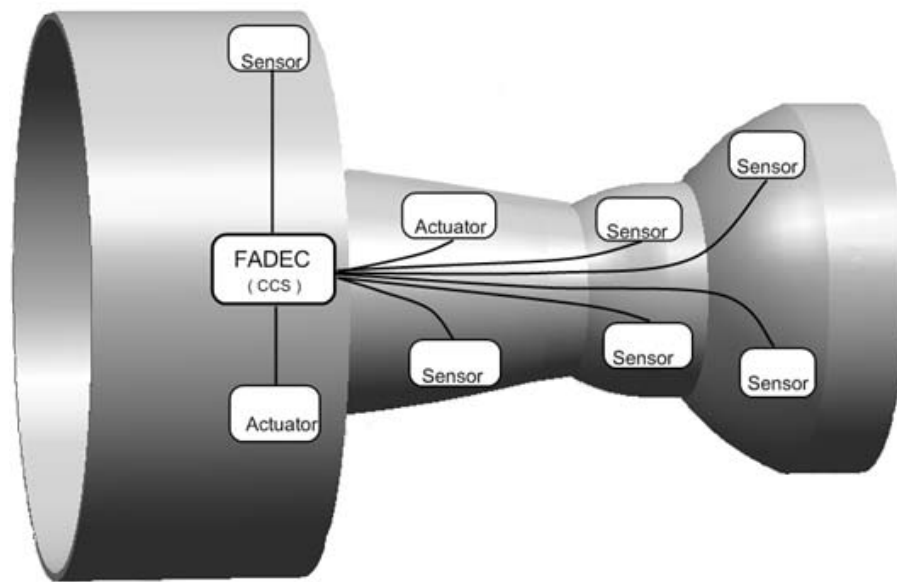
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## I. Introduction

Development of gas turbine engine control began in the early 1940s. Initially, engine control was executed using a hydro-mechanical governor for fuel metering. However, as the engines became more complex, the control task evolved to become more demanding and complex. With the advancement of solid state electronics, analog circuits were used for high level supervisory control, trim and other non-critical functions. Development in modern control theory, increased reliability, ease of maintenance, flexibility of control algorithms and advancement in digital electronics motivated use of digital control systems. This led to development of Full Authority Digital Engine Control (FADEC) which became the industry standard for commercial and military jet engines. In recent years, increasingly sophisticated electronics have been added to the engine control system for addressing the needs of increased performance, wider operability, and reduced life-cycle cost. Control systems are now designed to perform various other functions including engine diagnostics and health monitoring. This has driven the need for a new, advanced control system. Accordingly, a Distributed Engine Control Working Group (DECWG) was formed to study and develop a new Distributed Engine Control (DEC).<sup>1-2</sup>

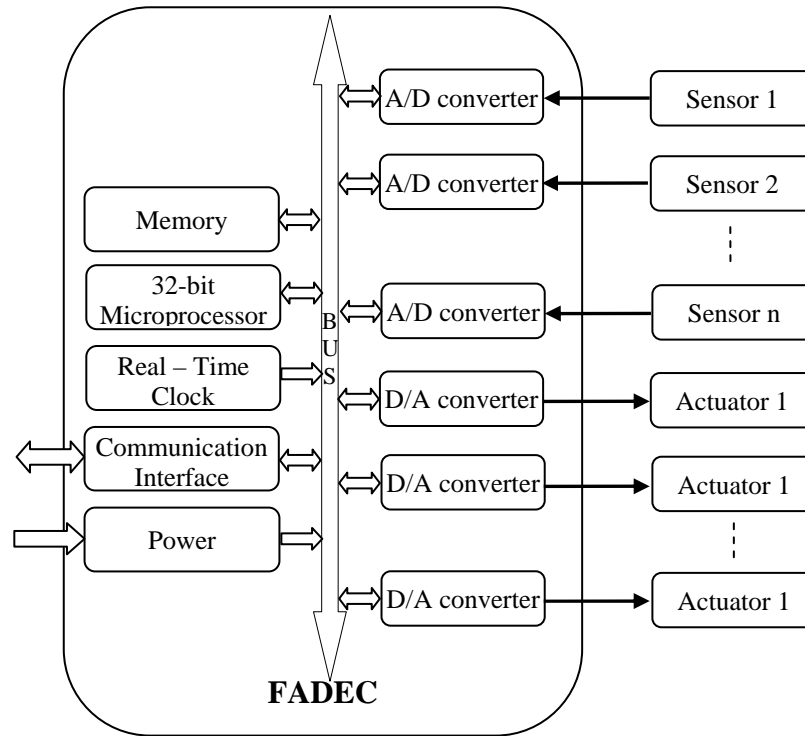
### A. FADEC based on centralized Control System (CCS) Architecture

In the traditional Centralized Control System (CCS) configuration, the centralized control processor handles all processing functions, including; the operating system, task scheduling, I/O, protection, communication, and control algorithms.<sup>3</sup> All computations are performed by a single controller and the control signals are transmitted to each individual actuator. The transmission of control signal is through individual analog connections with each sensor and actuator. FADEC houses the electronics required for data acquisition and signal conditioning. In order to provide required safety and reliability, dual channel communication links are used. Figure 1 shows an engine control system based on centralized architecture.



**Figure 1. FADEC based on centralized architecture**

The optimal location for the FADEC in order to reduce the wire harness length is near the combustion chamber. However, this is not practically possible as extra structural rigidity has to be added to protect it against high temperature and vibrations. Hence, the FADEC is placed on the aircraft engine fan case, which increases the wire harness length. As data communication is analog, the wire harness and connectors have to be shielded from noise and signal attenuations. This shielding increases the control system weight. The operational life of FADEC is one-third of the engine life. As the FADEC uses non-standard input/output interfaces, it is not easily upgradable. This increases obsolescence cost of engine.



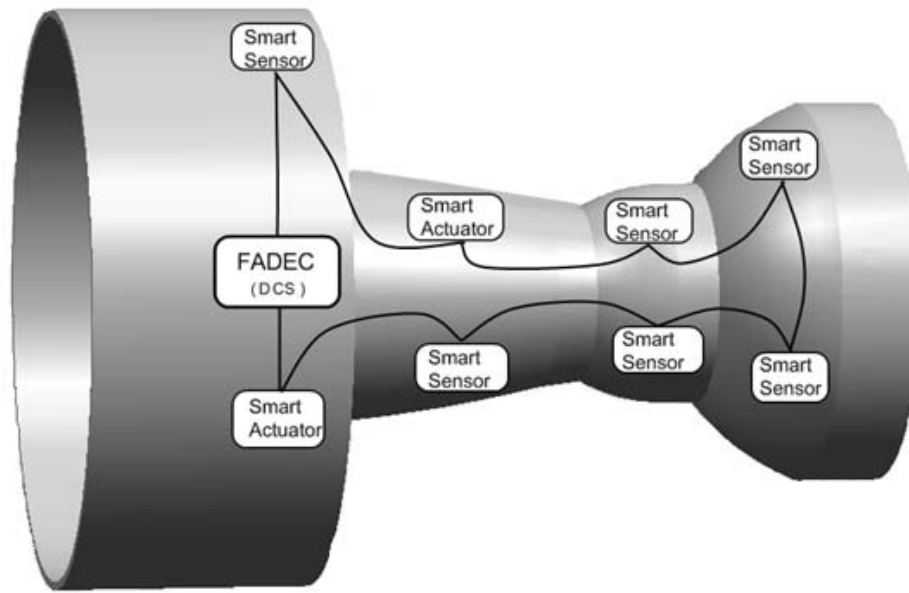
**Figure 2. Schematic diagram of current centralized FADEC**

Figure 2 illustrates point-to-point analog connections with sensors and actuators. We can see that while most of the processing is done by the FADEC, it is difficult to embed intelligent control directly into sensors and actuators. Also it is difficult to fault-check the electronic components of the FADEC, which increases engine down-time. Analog communication and of processing power limitation do prohibit advanced control algorithms.

Research is being carried out to make aircraft propulsion systems more intelligent, reliable, self-diagnostics, self-prognostics, self-optimizing and mission adaptable while also reducing the engine acquisition and maintenance cost. Advanced technologies like Blended Wing Body (BWB), Extreme Short Take Off and Landing (ESTOL), etc. require use of high performance intelligent engines. Incentives like reduction of weight, reduction in overall cost and increase in performance, led to the formation of Distributed Engine Control Working Group (DECWG). National Aeronautics and Space Administration, Glenn Research Center (NASA GRC) and industry came together through this group to study the current and future propulsion system requirements. They studied Distributed Engine Control (DEC) based on Distributed Control Systems (DCS), which had high benefits while adhering to the related control system and engine constraints. Distributed control is a mechanism for the proper implementation of systems engineering processes in engine systems based on open systems standards.

### **B. FADEC based on distributed architecture (DCS)**

In Distributed Engine Control, the functions of FADEC will be distributed at the component level. Each sensor/actuator is to be replaced by a smart sensor/actuator. These smart modules will include local processing capability to allow modular signal acquisition and conditioning, digital data bus communications and diagnostics and health management functionality. A serial communication network will be used to connect these smart modules with FADEC. With a use of such architecture, huge savings in weight, perhaps to the order of 50%, are expected.<sup>4</sup> There are no restrictions on the location of the DEC unit, therefore it can be placed at a location where it is subjected to less vibrations and hostile environment. This will reduce the design complexity reducing the weight and thermal constraints. Figure 3 shows an engine control system based on distributed architecture.



**Figure 3. FADEC based on Distributed architecture**

### **C. Benefits of Distributed Engine Control**

#### *1. Weight Reduction of propulsion control system*

Approximately, 50% of the electronic circuitry in a centralized FADEC is involved in processing of analog information. With DCS, estimated reductions in volume and weight of FADEC are on the order of 50%. This reduction in weight is assuming that the FADEC is engine mounted. However, due to digital communication, the FADEC can be mounted at a location in the aircraft where the conditions are less hostile, thus reducing the weight by a greater extent. A typical gas turbine engine has 15 sensed inputs (typically 5-wire) and seven controlled variables (typically 14-wire) which are dual redundant and shielded against electromagnetic and environmental effects. Replacing these point-to-point connections with serial communication wire harness could result in considerable savings in wire harness and connector weights.

#### *2. Modularity*

One of the main benefits of using DCS is the use of open system standards for its implementation. A modular system (i.e. smart sensor, smart actuator or even a FADEC), can be easily replaced or upgraded with a functionally equivalent component, without modifying the remaining system. This allows user customized design of a DEC. This modularity applies to both hardware and software.

#### *3. Obsolescence Reduction*

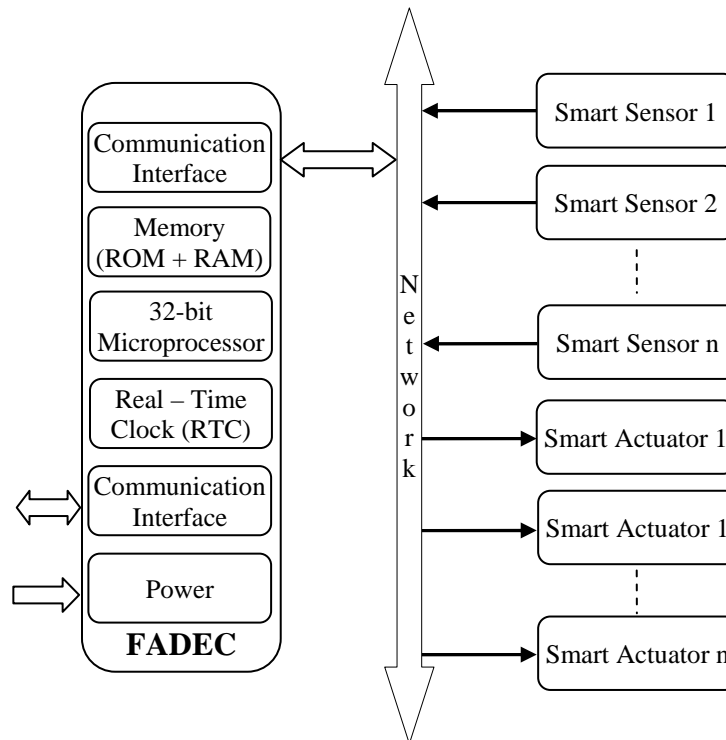
The Life cycle of a FADEC is around 8 years while engine life exceeds 20 years. As electronics improves, the FADEC has to be either redesigned; or old, obsolete electronic components have to be used. As a result it is very costly to redesign or replace FADEC for centralized control systems. However, due to DEC modularity, the amount of hardware which must be redesigned is minimized. Only the obsolete component can be replaced. This reduces the obsolescence costs.

#### 4. Improvement in Engine System Performance

Engine performance is judged in terms of emissions, noise, thrust efficiency, operability, time-on-wing, safety. A DEC will allow high-response adaptive control technologies (such as compressor stability control) to be implemented.

#### 5. Cost Reduction

Overall cost of the engine can be subdivided – into acquisition cost, operating cost and maintenance cost. A DEC is believed to have a positive impact on all of the above costs. As a DEC is based on open system architecture, there is commonality among the system components. This lowers the development cost and in turn the acquisition cost. Weight reduction increases fuel efficiency. This, along with the high rate of system availability, decreases the operating cost. Due to distributed architecture, maintenance cost is also reduced. Faulty components can be easily located and replaced, decreasing the maintenance time.



**Figure 4. Schematic Diagram for FADEC based on DCS**

The distributed control approach will be inherently more powerful, flexible, and scalable than a centralized control approach. However, there are major technical challenges to the realization of DEC. High temperature electronics, selection of appropriate communication architecture, and partitioning of the centralized controller are some to name a few. Silicon-On-Insulator (SOI) is one of the promising high temperature electronics technologies. The operational temperature for SOI is in the range of 225 °C to 250 °C. For a temperature higher than this, the technology is still in development stage. It is projected that use of Silicon Carbide electronics will enable use of electronics at temperatures above 600 °C. In DEC, the appropriate selection of communication architecture is very important as the performance of the DEC will be dependent on the performance of the communication network. The network must have sufficient bandwidth and latency to enable closed loop control. It must also be robust to accommodate the safety and critical functions. In the following section we give the methodology for design of DEC.

## II. Methodology for design of DEC

We can have two design approaches to DCS. In first approach, we can partition the centralized controller to achieve optimum performance of subsystems, and then select the communication architecture. In the second approach, we can first select the communication architecture which is desired for the distributed system, and then partition the controller given the communication constraints. In this approach, we can make use of commercial off-the-shelf components, reducing the design time and cost. Hence, this approach is followed in this research. Accordingly, the first task is to select the appropriate communication architecture (protocol) for safety-critical turbine engine.

### A. Communication Architectures

For safety-critical DCS, there is a clear preference for time-triggered protocols over the event-driven protocols. Time-triggered protocols offer high level of reliability with fault-tolerance. These architectures ensure that maximum bus loading stays at prescribed levels and also provides efficiency, determinism and partitioning. Some of the off-the-shelf time triggered protocols are MIL-STD-1553, SAFEBus, FlexRay, CAN, SPIDER, Time Triggered Protocol TTP/C and IEEE 1394b/Firewire. Out of these architectures, TTP/C, developed by University of Vienna, has clear advantages over the others.<sup>5-6</sup> Some of the requirements of communication architecture for DCS are that it should support fault detection, isolation, and recovery (FDIR) and health monitoring; should be highly modular, with high reliability, should be easy to maintain and finally should have low overall cost. All these requirements are best met by TTA. The TTA developed by University of Vienna uses a time-triggered communication protocol called TTP/C. TTP/C is specially designed for the safety critical, hard real-time distributed control. As the protocol is master-less, it enables the communication between nodes to continue even when there is a failure in any of the nodes. Also the communication rate provided by the current TTP/C meets the demand for real-time control requirement of safety critical vehicle subsystems. Along with high transmission rate, TTP/C has high data efficiency, error detection with short latency, a fault-tolerant clock synchronization service, and distributed redundancy management. This architecture can tolerate multiple faults and high degree of temporal predictability. One of the main advantages of TTP/C is the availability of the commercial off-the-shelf components. This reduces the overall cost of implementation of TTP/C making it a cost effective solution. Accordingly, the use of TTP/C for DEC is suggested.<sup>4</sup> Considering TTP/C as the communication architecture, we will now focus on the design aspects of NCS.

### B. Networked Control Systems (NCS)

DCS have multiple, independent processes and the output of any distributed process is based on the quality of input data which must be transmitted accurately and without any delay. Distributed Engine Control Systems can be viewed as a Networked Control System (NCS) with distributed sensors and actuators. Here, the control loops are closed through a real-time communication network. This addition of a communication network in the system introduces complexity in the design. There are various factors introduced as a result of a communication network. They include network induced delay, packet dropouts, and bandwidth constraints, which have to be considered for ensuring desired functionality of the NCS.<sup>7-8</sup>

#### 1. Network-induced Time Delay

Transmission of continuous time signal over a network consists of data sampling, data encoding, data transmission and data decoding. The total network induced time delay consists of network access delay and network transmission delay. Network access delay is the time taken for the shared network to accept data. Network transmission delay can be further subdivided into sensor-to-controller delay and controller-to-actuator delay. Network transmission delay can be either constant, time varying or random depending on the network congestion and channel quality. In the selected TTP/C architecture, the likelihood of time delays is reduced by using clock synchronization, transmission window timing and group membership.<sup>9</sup> In this paper, it is assumed that time delays are less than the predetermined threshold. Whenever the delay is more than threshold, the packet is dropped, and a new packet is transmitted.

#### 2. Constraint on Channel Bandwidth

The capacity of the communication network to carry a finite amount of information per unit amount of time is known as channel bandwidth. The current available hardware supports 25 Mbit/s synchronous and 5 Mbit/s



asynchronous transmission and transmission rates of 1 Gb/s have been tested. The transmission rate provided by the current TTP/C meets the demand for real-time control requirement of safety critical vehicle subsystems.

### 3. Packet Dropouts

Packet dropouts occur due to packet collisions or node failures. Although the time triggered protocol ensures that packet collisions do not occur (as each node can transmit/receive data only during the predetermined time slot) the network is still subjected to node failures. When a node failure occurs, (instead of repeating retransmission attempts) it is advantageous to drop the old packet and transmit a new one. In single packet transmission, packet drop can be viewed as loss of all of the state information, and in multiple packet transmission, as the loss of partial state information. Although feedback control systems can tolerate packet dropouts to some extent, it is important to analyze the system stability and performance under packet dropouts.

The membership mechanism of TTP is capable of detecting any kind of communication fault that is not already detected and handled by other means. These communication faults include transmission and reception faults. If a node fails to transmit (which is typically due to noise during the transmission), it is removed from the membership list and is not allowed to transmit data.<sup>9</sup> Immediate retransmission for this node is not allowed and it can retry transmission in the next round. This results in packet dropouts. Hence, for communication architectures implemented using TTP/C, it is important to consider stability and performance of the system under packet dropouts. In this paper, we analyze the effect of packet dropouts on the stability of the system considering single-packet transmission of plant inputs and outputs.

## III. Decentralized Distributed Engine Control Systems

### A. Decentralized Distributed Full Authority Engine Control (D<sup>2</sup>FADEC).

Centralized design accounts for subsystem interactions and provides a baseline for the integrated system. The coupling between propulsion systems and airframe is highly significant; therefore the propulsion control system and flight control system cannot be designed separately. Hence, during design of the FADEC, it was necessary to follow a centralized approach. However, the performance and robustness of a higher order centralized control system is less than that of the lower order subsystems. Also the centralized approach does not consider the system nonlinearities of subsystems. Controllers based on the decentralized framework allows us to consider the interactions between the subsystems and at the same time optimizes subsystem performance. This approach provides improved component fault-prognostics and fault-tolerance while reducing the processing complexity. The use of a decentralized multi-variable control algorithm for turbine engine control is well discussed in the engine control community. It has been shown that the use of decentralized control structure not only improves the performance of gas turbine engine, but also reduces the number of controller design operating points. Also the controller is made more robust and the system remains stable in presence of soft and/or hard failures. Hence, we propose a controller based on Decentralized Distributed Control Architecture, namely *Decentralized Distributed Full Authority Engine Control (D<sup>2</sup>FADEC)*.<sup>10-11</sup>

### B. Mathematical model for D<sup>2</sup>FADEC

A gas turbine engine essentially consists of interconnected subsystems. By identifying these subsystems, we can take advantage of the structural features of the system in control design. If we are not able to identify the subsystems, we can decompose the entire system into “mathematical” subsystems. These kinds of partitions of dynamic subsystems were termed as *physical* and *mathematical* decompositions and were studied by Siljak.<sup>12</sup>

Let us consider a linear system consisting of  $N$  interconnected subsystems

$$\mathbf{S}: \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (1)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

which can be partitioned into

$$\mathbf{S}: \dot{\mathbf{x}}_i = \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_i\mathbf{u}_i + \sum_{j=1}^N (\mathbf{A}_{ij}\mathbf{x}_j + \mathbf{B}_{ij}\mathbf{u}_j) \quad (2)$$

$$y_i = C_i x_i + \sum_{j=1}^N C_{ij} x_j \quad i \in N$$

where  $x_{(k+1)_i} \in \mathbb{R}^{n_i}$ ,  $u_{(k)_i} \in \mathbb{R}^{m_i}$ ,  $y_{(k+1)_i} \in \mathbb{R}^{l_i}$  are the state, input and output of subsystems,  $S_i$

A more compact notation for the above system is

$$\mathbf{S}: \dot{x} = A_D x + B_D u + A_C x + B_C u \quad (3)$$

$$y = C_D x + C_C x \quad i \in N$$

where,

$$A_D = \text{diag}\{A_1, A_2, \dots, A_N\}$$

$$B_D = \text{diag}\{B_1, B_2, \dots, B_N\}$$

$$C_D = \text{diag}\{C_1, C_2, \dots, C_N\}$$

and coupling block matrices are

$$A_C = (A_{ij}), \quad B_C = (B_{ij}), \quad C_C = (C_{ij})$$

Consider full state feedback and a linear constant control law for the original plant  $S$  can be chosen as

$$u = -Ky \quad (4)$$

$$K = K_D + K_C$$

The closed loop system becomes,

$$\widehat{\mathbf{S}}: \dot{x} = (A_D - B_D K_D - B_C K_D)x + (A_C - B_C K_C - B_D K_C)x \quad (5)$$

$$\widehat{\mathbf{S}}: \dot{x} = \hat{A}_D x + \hat{A}_C x$$

where,

$$\hat{A}_D = (A_D - B_D K_D - B_C K_D)$$

$$\hat{A}_C = (A_C - B_C K_C - B_D K_C)$$

#### 1. Designing the local controller, $K_D$

Now we shall consider the selection of local controller gains to exponentially stabilize the overall system to prescribed degree. As we want the subsystems to have optimum performance, we first assume that there are no interactions between the subsystems, i.e.  $A_C = 0$

We can use any controller design method to find local controller gains. In this paper, we will find the optimal gain matrix,  $K_D$ , by solving the discrete time algebraic Riccati equation for each local subsystem. The discrete time algebraic Riccati equation is given as

$$A_D^T X_i A_{D_i} - X_i - A_D^T X_i B_{D_i} (B_D^T X_i B_{D_i} + R_{D_i})^{-1} B_D^T X_i A_{D_i} + Q_{D_i} = 0 \quad (6)$$

$$K_{D_i} = (B_D^T X_i B_{D_i} + R_{D_i})^{-1} B_D^T X_i A_{D_i} \quad (7)$$

$$u^l = -K_D y \quad (8)$$

## 2. Designing the global controller, $K_C$

We select global gain matrix,  $K_C$ , such that  $\hat{A}_C \triangleq 0$ , which corresponds to reducing the effect of the interconnections.<sup>13</sup> This can be done by selecting

$$K_c = B^{\#}A_c \quad (9)$$

$$u^g = -K_c y \quad (10)$$

If matrix  $B$  is of full column rank, then the interactions will be completely nullified and  $\hat{A}_C = 0$ . If  $B$  is not of full column rank, then  $\hat{A}_C \triangleq 0$  and we will have to ensure that the closed loop system remains stable. For this we consider  $\hat{A}_C$  as unstructured perturbations and use the results obtained in Ref.14 to determine system stability.

The system, (5) is stable if the following condition is satisfied.

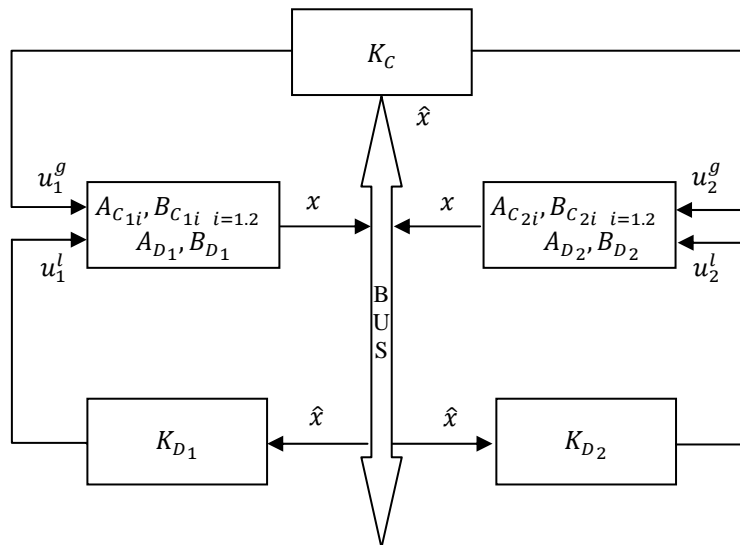
$$\sigma_{max}(\hat{A}_C) < -\sigma_{max}(\hat{A}_D) + \left( [\sigma_{max}(\hat{A}_D)]^2 + \frac{\sigma_{min}(Q)}{\sigma_{max}(P)} \right)^{1/2} \quad (11)$$

Where,  $Q = I$  and  $P$  is solution of discrete time Lyapunov Equation solved for  $\hat{A}_D$

### C. Mathematical model for D<sup>2</sup>FADEC considering packet dropouts

Consider architecture as shown in Fig. 5. Both the local controller and global controller are connected through the network. These controllers can be physically separated from each other or can be housed together. For simplicity, consider a plant having 2 subsystems. We now will study the effect of packet dropouts on the performance of the closed loop system.

The packet dropping process of a communication network in discrete time can be modeled as either an independent and identically distributed (i.i.d.) Bernoulli random process or a Markov chain. Hu and Yan studied the effect of packet dropping in Ref. 15. They modeled the packet dropping of a communication network as an i.i.d Bernoulli process and studied the stability of a discrete-time NCS with static state feedback where the PDP of the communication network is bounded by a known upper bound. We will now extend their results for a controller under decentralized framework.

Figure 5. D<sup>2</sup>FADEC with distributed Local and Global Controller

The network is assumed to be modeled by

$$\hat{x}(k) = \xi(k) x(k) \quad (12)$$

where,  $\xi(k)$  is the independent and identically distributed (i.i.d.) Bernoulli random process.  $\xi(k)$  can be either 0 or 1 at any time instant  $k$ . Value 0 indicates the packet is lost during transmission while value 1 indicates successful transmission of packet. The probability of  $\xi(k) = 0$  is termed as Packet Dropping Probability (PDP) and is a measure of the reliability of the network.

Now, when the packet is dropped,  $\xi(k) = 0$  i.e.,  $u(k) = 0$ . If the packet is transmitted successfully  $\xi(k) = 1$  and  $\hat{x}(k) = x(k)$ .

Stability Condition for NCS: The networked plant with PDP equal to the constant  $\alpha$  is ms-stabilized (stabilized in mean-square sense) by the controller if and only if the following condition holds<sup>16</sup>:

$$\rho[\alpha A \otimes A + (1 - \alpha)\hat{A} \otimes \hat{A}] < 1 \quad (13)$$

where,  $\otimes$  is the Kronecker product and  $\rho(\cdot)$  is the spectral radius of the matrix and  $\hat{A} = A - BK$

Hu and Yan introduced a term known as Packet Dropping Margin (PDM) which is defined as the largest positive bound  $\alpha$  such that the system is ms-stable for any PDP less than  $\alpha$ . They gave a formula to find PDM which is as follows.<sup>15</sup>:

If the NCS is nominally stable, then,

$$PDM = \frac{1}{\mu(V)} \quad (14)$$

where,

$$V = \begin{bmatrix} (S \otimes \tilde{S} + \tilde{S} \otimes S)(I - S \otimes S)^{-1} & \tilde{S} \otimes \tilde{S} \\ (I - S \otimes S)^{-1} & 0 \end{bmatrix} \quad (15)$$

$$S = \hat{A} \otimes \hat{A}, \quad \tilde{S} = A \otimes A - \hat{A} \otimes \hat{A}$$

Also, they gave a lower bound for PDM, which is dependent on the  $K_2(\hat{A})$

$$PDM \geq \frac{1 - \rho^2(\hat{A})}{K_2^2(\hat{A})\|A\|_2^2 - \rho^2(\hat{A})} \quad (16)$$

It is observed that PDM is inversely proportional to  $K_2(\hat{A})$ . Hence, Hu and Yan proposed an algorithm to maximize PDM by minimizing  $K_2(\hat{A})$ .

Now we will study two types of system, one where each subsystem has independent control and one where each control input affects two or more subsystems.

#### 1. Case I: $B_C = 0$

When  $B_C = 0$ , closed loop system reduces to following form

$$\hat{\mathbf{S}}: \dot{x} = (A_D - B_D K_D)x + (A_C - B_D K_C)x \quad (17)$$

or in compact form,

$$\widehat{\mathbf{S}}: \dot{x} = \hat{A}_D x + \hat{A}_C x$$

In order to reduce the effect of interactions, we make  $\hat{A}_C = 0$  by selecting

$$K_C = B_D^\# A_C \quad (18)$$

**Example 1:** In order to compare decentralized and centralized controller, we consider an example from literature.<sup>17</sup>

$$A = \begin{bmatrix} 1.2 & 0.1 & -0.3 \\ 0.5 & -0.2 & -0.3 \\ -2.5 & 1.8 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And the desired nominal closed loops as  $-0.7053, 0.4777 \pm 0.4535i$

Algorithm proposed by Hu-Yan is used to find a feedback gain which maximizes PDM. This gain yields a PDM of 0.3449 and  $K_2(\hat{A}) = 1.0709$

We design a decentralized controller for above system,

$$K_D = \begin{bmatrix} 0.7223 & 0.5535 & 0 \\ 0.0930 & -1.3554 & 0 \\ 0 & 0 & 0.7053 \end{bmatrix}, \quad K_C = \begin{bmatrix} 0 & 0 & -0.3 \\ 0 & 0 & -0.6 \\ -2.5 & 1.8 & 0 \end{bmatrix}$$

This gain yields  $PDM_{DS} = 0.39$ ,  $K_2(\hat{A}) = 1.0708$

We observe that there is significant improvement in PDM for the same nominal closed loop poles. We also observe that with  $K_2(\hat{A})$  obtained with the use of a controller in decentralized framework, is lower than  $K_2(\hat{A})$  obtained by Hu and Yan algorithm. This shows that PDM is dependent on the structure of  $\hat{A}$ . As we no longer have to solve the minimization problem, the computational effort and time is significantly reduced. Hence, use of decentralized controller gives a lower  $K_2(\hat{A})$ , with higher PDM and with less computational effort.

2. *Case II:*  $B_C \neq 0$

For this case, the closed loop system becomes,

$$\widehat{\mathbf{S}}: \dot{x} = (A_D - B_D K_D)x + (A_C - B_C K_D - B_C K_C - B_D K_C)x \quad (19)$$

$$\dot{x} = (A_D - B_D K_D)x + (\widetilde{A}_C - B K_C)x$$

where,  $\widetilde{A}_C = A_C - B_C K_D$

or in compact form,

$$\widehat{\mathbf{S}}: \dot{x} = \hat{A}_D x + \hat{A}_C x$$

In order to reduce the effect of interactions, we make  $\hat{A}_C \cong 0$  by selecting

$$K_C = B^\# \widetilde{A}_C \quad (20)$$

### Example 2:

We now study the effect of packet dropouts for a F100 engine under decentralized framework. The model is obtained from Ref. 18. The continuous time model is converted into a discrete time model with sampling time of 0.01 s.

Let the networked plant (1)-(4) be as shown below.

$$A = \begin{bmatrix} 0.9598 & 0.0365 & -4.6317 & 0.0608 & 0.0482 & -0.0332 \\ 0.0003 & 0.9708 & -0.5745 & 0.0012 & 0.0199 & 0.1221 \\ 0.0003 & 0.0001 & 0.9556 & 0.0000 & 0.0014 & 0.0010 \\ 0.0085 & -0.0204 & -2.5621 & 0.6065 & -0.0057 & -0.0920 \\ 0.0004 & -0.0009 & -0.1158 & -0.0156 & 0.9799 & -0.0042 \\ 0.0000 & 0.0000 & -0.0059 & 0.0000 & 0.0001 & 0.9934 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0108 & 1.6642 & -0.9764 & 0.0851 \\ 0.0091 & -0.3464 & -0.0142 & -0.5932 \\ -0.0001 & -0.7790 & 0.0116 & 0.0036 \\ 0.0444 & 0.2358 & -0.1646 & 0.3287 \\ 0.0020 & 0.0103 & -0.0074 & 0.0148 \\ 0.0001 & -0.0008 & -0.0003 & 0.0007 \end{bmatrix}$$

$$C = I_{6 \times 6}$$

$$D = 0$$

As the physical subsystems are not known, we will study the effect of mathematical partitioning on PDM for the NCS as shown above. The local controller gain,  $K_D$  is obtained using pole placement technique for desired closed loop poles  $0.02 \pm 0.5i$ , 0.2, 0.7, 0.9801 and 0.9936. Global controller,  $K_C$  was calculated using Eqn. 20.

Type of Partitioning	PDM	$K_2(\hat{A})$
Centralized architecture	0.1955	4.9792
$A_1^{2 \times 2}, B_1^{2 \times 1}, A_2^{4 \times 4}, B_2^{4 \times 3}$	0.0117	5.4766e+004
$A_1^{2 \times 2}, B_1^{2 \times 2}, A_2^{4 \times 4}, B_2^{4 \times 2}$	1.0386	4.9759
$A_1^{2 \times 2}, B_1^{2 \times 3}, A_2^{4 \times 4}, B_2^{4 \times 1}$	1.0394	2.0832e+004
$A_1^{3 \times 3}, B_1^{3 \times 1}, A_2^{3 \times 3}, B_2^{3 \times 3}$	1.1140e-007	2.5205e+010
$A_1^{3 \times 3}, B_1^{3 \times 2}, A_2^{3 \times 3}, B_2^{3 \times 2}$	0.3204	4.9717
$A_1^{3 \times 3}, B_1^{3 \times 3}, A_2^{3 \times 3}, B_2^{3 \times 1}$	1.0338	4.9688
$A_1^{4 \times 4}, B_1^{4 \times 1}, A_2^{2 \times 2}, B_2^{2 \times 3}$	3.7273e-008	3.5374e+009
$A_1^{4 \times 4}, B_1^{4 \times 2}, A_2^{2 \times 2}, B_2^{2 \times 2}$	0.3285	77.4584
$A_1^{4 \times 4}, B_1^{4 \times 3}, A_2^{2 \times 2}, B_2^{2 \times 1}$	0.3835	35.7265

**Table 1. Table to show dependence of PDM on the partitioning**

From the above table, it can be observed that centralized controller gives a PDM of 0.1955. The data transmission efficiency of TTP/C being 70%-90%, it has a PDM of 0.3. The PDM for centralized controller being less than that of TTP/C, FADEC based on centralized architecture can become unstable when DCS is implemented using TTP/C. However, we observe that the PDM depends on the system partitioning and by selecting a suitable

system partition; we can obtain a large PDM. For the above example, we select partition given as  $A_1^{2 \times 2}, B_1^{2 \times 2}, A_2^{4 \times 4}, B_2^{4 \times 2}$  as it gives largest PDM with small  $K_2(\hat{A})$ .

If the nominal closed loop poles are not given, we can use Eqn. 6 to obtain local controller gain,  $K_D$ , which can be thought of optimizing the subsystems. Such physical partitioning can be done only when the subsystem models are known. Physical partitioning was also performed for the same engine model. It was observed that PDM for physical partitioning is also dependent on the system partitioning and the same partition,  $A_1^{2 \times 2}, B_1^{2 \times 2}, A_2^{4 \times 4}, B_2^{4 \times 2}$ , gives largest PDM.

#### IV. Conclusions

Advanced future propulsion control demands for an intelligent, fault tolerant systems and it is necessary develop a new control system. The benefits of Distributed Controls Systems are beginning to be recognized in the engine community. In this paper, use of TTP/C as a communication architecture is highlighted. Distributed Engine Control under Decentralized Framework is proposed for achieving high performance, low cost and fault tolerant control systems. Based on this framework, *Decentralized Distributed Full Authority Engine Control (D<sup>2</sup>FADEC)* is introduced and a mathematical model consisting of two- level controller structure is formed. Two such mathematical models of DCS are analyzed for performance under packet dropouts. Using the concept of PDM, it is shown that use of decentralized architecture is advantageous over centralized architecture. In first case, it is shown that the PDM is dependent on the structure of the closed-loop matrix; nullifying the effect of interactions can therefore result in a significant improvement of PDM. We also observe that PDM is less dependent on condition number and more dependent on the subsystem interactions. In the second case, this property is exploited and the centralized system is decoupled to ensure that the effects of interactions are reduced. We apply this theory to a F100 engine model available in literature and observe that the PDM also depends on the system partitioning. By selecting a suitable mathematical partition, we can obtain a large PDM, given that the system has prescribed nominal closed-loop poles. The same results can be extended to case of where the control input is also subject to packet dropouts.

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